

## **AMENDMENTS TO THE SPECIFICATION**

### **Please amend the paragraph starting on page 1, line 19 as follows:**

The subject matter of this application is related to the subject matter in a co-pending non-provisional application by the same inventors as the instant application and filed on the same day as the instant application entitled, "Applying Term Consistency to an Equality Constrained Interval Global Optimization Problem," having serial number ~~TO BE ASSIGNED~~ 10/017,573, and filing date ~~TO BE ASSIGNED~~ 13 December 2001 (Attorney Docket No. SUN-P6445-SPL).

### **Please amend the paragraph starting on page 5, line 17 as follows:**

One embodiment of the present invention provides a system that solves a global optimization problem specified by a function  $f$  and a set of inequality constraints  $p_i(\mathbf{x}) \leq 0$  ( $i=1, \dots, m$ ), wherein  $f$  is a scalar function of a vector  $\mathbf{x} = (x_1, x_2, x_3, \dots, x_n)$ . The system operates by receiving a representation of the function  $f$  and the set of inequality constraints, and then storing the representation in a memory within the computer system. Next, the system applies term consistency to the set of inequality constraints over a ~~subbox-sub-box~~  $\mathbf{X}$ , and excludes any portion of the ~~subbox-sub-box~~  $\mathbf{X}$  that violates any member of the set of inequality constraints.

### **Please amend the paragraph starting on page 5, line 26 as follows:**

In one embodiment of the present invention, the system additionally linearizes the set of inequality constraints to produce a set of linear inequality constraints with interval coefficients. Next, the system preconditions the set of linear inequality constraints using additive linear combinations to produce a preconditioned set of linear inequality constraints. The system then applies term consistency to the set of preconditioned linear inequality constraints over the

~~sub-box~~ ~~sub-box~~  $X$ , and excludes any portion of the ~~sub-box~~ ~~sub-box~~  $X$  that violates set of preconditioned linear inequality constraints. In a variation on this embodiment, the system additionally keeps track of a least upper bound  $f\_bar$  of the function  $f(x)$  in the feasible region, and includes  $f(x) \leq f\_bar$  in the set of inequality constraints prior to linearizing the set of inequality constraints. In a variation on this embodiment, the system removes from consideration any inequality constraints that are not violated by more than a predetermined amount for purposes of applying term consistency prior to linearizing the set of inequality constraints.

**Please amend the paragraph starting on page 6, line 14 as follows:**

In one embodiment of the present invention, performing the interval global optimization process involves keeping track of a least upper bound  $f\_bar$  of the function  $f(x)$  in the feasible region, and removing from consideration any ~~sub-box~~ ~~sub-box~~ for which  $f(x) > f\_bar$ . It also involves applying term consistency to the inequality  $f(x) \leq f\_bar$  over the ~~sub-box~~ ~~sub-box~~  $X$ , and excluding any portion of the ~~sub-box~~ ~~sub-box~~  $X$  that violates the inequality.

**Please amend the paragraph starting on page 6, line 20 as follows:**

In one embodiment of the present invention, if the ~~sub-box~~ ~~sub-box~~  $X$  is strictly feasible ( $p_i(X) < 0$  for all  $i=1, \dots, n$ ), performing the interval global optimization process involves determining a gradient  $g(x)$  of the function  $f(x)$ , wherein  $g(x)$  includes components  $g_i(x)$  ( $i=1, \dots, n$ ). Next, the system removes from consideration any ~~sub-box~~ ~~sub-box~~ for which  $g(x)$  is bounded away from zero, thereby indicating that the ~~sub-box~~ ~~sub-box~~ does not include an unconstrained local extremum. The system also applies term consistency to each component  $g_i(x)=0$  ( $i=1, \dots, n$ ) of  $g(x)=0$  over the ~~sub-box~~ ~~sub-box~~  $X$ , and excludes any portion of the ~~sub-box~~ ~~sub-box~~  $X$  that violates a component.

**Please amend the paragraph starting on page 7, line 3 as follows:**

In one embodiment of the present invention, if the ~~sub-box~~ sub-box  $\mathbf{X}$  is strictly feasible ( $p_i(\mathbf{X}) < 0$  for all  $i=1, \dots, n$ ), the system performs the interval global optimization process by determining diagonal elements  $H_{ii}(\mathbf{x})$  ( $i=1, \dots, n$ ) of the Hessian of the function  $f(\mathbf{x})$ , and removing from consideration any ~~sub-box~~ sub-box for which a diagonal element of the Hessian is always negative, indicating that the function  $f$  is not convex and consequently does not contain a global minimum within the ~~sub-box~~ sub-box. The system also applies term consistency to each inequality  $H_{ii}(\mathbf{x}) \geq 0$  ( $i=1, \dots, n$ ) over the ~~sub-box~~ sub-box  $\mathbf{X}$ , and excludes any portion of the ~~sub-box~~ sub-box  $\mathbf{X}$  that violates the inequalities.

**Please amend the paragraph starting on page 7, line 12 as follows:**

In one embodiment of the present invention, if the ~~sub-box~~ sub-box  $\mathbf{X}$  is strictly feasible (that is  $p_i(\mathbf{X}) < 0$  for all  $i=1, \dots, m$ ), then the system uses the interval Newton method to find a box  $\mathbf{X}$  that contains a stationary point  $\mathbf{y}$  where the gradient of  $f$ ,  $\mathbf{g}(\mathbf{y}) = 0$ . This involves forming and solving the system of equations  $\mathbf{M}(\mathbf{x}, \mathbf{X})(\mathbf{y} - \mathbf{x}) = \mathbf{r}(\mathbf{x})$  for the bounding box  $\mathbf{Y}$  on  $\mathbf{y}$ : where  $\mathbf{M}(\mathbf{x}, \mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x}, \mathbf{X})$ ;  $\mathbf{J}(\mathbf{x}, \mathbf{X})$  is the Jacobian of the objective function  $f$  expanded about the point  $\mathbf{x}$  over the ~~sub-box~~ sub-box  $\mathbf{X}$ ;  $\mathbf{B}$  is the approximate inverse of the center of  $\mathbf{J}(\mathbf{x}, \mathbf{X})$ ; and  $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{g}(\mathbf{x})$ . The system also applies term consistency to each equality  $(\mathbf{B}\mathbf{g}(\mathbf{x}))_i = 0$  ( $i=1, \dots, n$ ) for each variable  $x_i$  ( $i=1, \dots, n$ ) over the ~~sub-box~~ sub-box  $\mathbf{X}$ , and excludes any portion of the ~~sub-box~~ sub-box  $\mathbf{X}$  that violates an equality.

**Please amend the paragraph starting on page 8, line 11 as follows:**

In one embodiment of the present invention, applying term consistency involves symbolically manipulating an equation within the computer system to solve for a first term,  $g(x_j)$ , thereby producing a modified equation  $g(x_j) = h(\mathbf{x})$ ,

wherein the first term  $g(x_j)$  can be analytically inverted to produce an inverse function  $g^{-1}(y)$ . Next, the system substitutes the ~~sub-box~~  $X$  into the modified equation to produce the equation  $g(X'_j) = h(X)$  and then solves for  $X'_j = g^{-1}(h(X))$ . The system then intersects  $X'_j$  with the interval  $X_j$  to produce a new ~~sub-box~~  $X^+$ , which contains all solutions of the equation within the ~~sub-box~~  $X$ , and wherein the size of the new ~~sub-box~~  $X^+$  is less than or equal to the size of the ~~sub-box~~  $X$ .

**Please amend the paragraph starting on page 24, line 25 as follows:**

In various previous steps, gaps may have been generated in components of  $X$ . If so, the system merges any of these gaps that overlap. The system then splits  $X$ , and places the resulting ~~sub-boxes~~ in  $L_I$  and goes to step 703 (step 742).

**Please amend the paragraph starting on page 25, line 1 as follows:**

If all of the initial box  $X^{(0)}$  is deleted by our process, then we have proved that every point in  $X^{(0)}$  is infeasible. Suppose that every point in  $X^{(0)}$  is infeasible. Our process may prove this to be the case. However, we delete a ~~sub-box~~ of  $X^{(0)}$  only if it is *certainly* infeasible. Rounding errors and/or dependence may prevent us from proving certain infeasibility of an infeasible ~~sub-box~~. Increased wordlength can reduce rounding errors and decreasing  $\epsilon_X$  can reduce the effect of dependence by causing ~~sub-boxes~~ to eventually become smaller. However, neither effect can completely be removed.

**Please amend the paragraph starting on page 26, line 11 as follows:**

Experience has shown that efficiency is enhanced if the ~~sub-box~~  $X$  to be processed is chosen to be the one for which  $\inf(f(X))$  is smallest among all candidate ~~sub-boxes~~. This tends to cause a smaller value of  $f\_bar$  to be

computed early in the algorithm. Therefore, we return to step 703 to choose a  
new ~~subbox~~sub-box whenever the current box has substantially changed.